The elastic pd scattering analyzing powers and spin correlation coefficients at $\mathsf{E}_p^{lab}=135$ and 200 MeV: Three-nucleon force and relativistic effects

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Abstract. The rich set of data for analyzing powers and spin correlation coefficients in pd elastic scattering at $E_p^{lab} = 135$ and 200 MeV taken at IUCF has been compared to theoretical predictions based on modern nuclear forces. To this aim the three-nucleon Faddeev equations have been solved with standard nucleon-nucleon potentials (AV18, CD Bonn, NijmI and II) alone or combined with the 2π -exchange Tucson-Melbourne three-nucleon force. For the AV18 potential also the Urbana IX three-nucleon interaction has been used. For some spin observables the addition of these three-nucleon forces improves the description of data whereas in other cases the description deteriorates. The relativistic effects for all studied spin observables have been investigated and found to be small. The conclusion is that significant parts of the three-nucleon force are missing.

PACS. 21.45.+v Few-body systems – 24.70.+s Polarization phenomena in reactions – 25.10.+s Nuclear reactions involving few-nucleon systems

1 Introduction

The study of three-nucleon force (3NF) properties is a hot topic of present-day investigations in few-nucleon systems. Two-nucleon forces alone are insufficient to describe binding energies. This has been clearly demonstrated with the advent of the modern nucleon-nucleon (NN) potentials, e.g. AV18 [1], CD Bonn [2], NijmI and II [3], which reproduce the existing NN data set up to about the pion threshold with high precision. Using those forces alone leads to clear-cut underbinding for ³He and ³H nuclei. That is generally seen as the most obvious experimental signature of 3NFs. The available 3NF models, such as the 2π -exchange Tucson-Melbourne (TM) [4] or Urbana IX [5] interactions, have been applied in order to check if they are able to improve the description of the data. This approach turned out to be successful to account for the underbinding of the very light nuclei [6-8]. The missing binding of ³H and 3 He of about 0.5–1 MeV was removed when the abovementioned standard NN potentials were supplemented by the (properly adjusted) TM or Urbana IX 3NFs [6,7]. For

light nuclei with a larger number of nucleons also a significant improvement of binding has been obtained when the Urbana IX 3NF has been added to the AV18 NN potential [8]. However, the remaining underbinding for A > 4 nuclei, which increases with increasing mass number A, indicates that some terms in Urbana IX are missing [8]. Indeed, augmenting the Hamiltonian by the Illinois 3NFs, based on three-pion exchanges with intermediate Δ 's, improves the description of light nuclei spectra [9,10].

Additional impetus in those studies came with the analyses of 3N elastic scattering and breakup reactions. The numerous cases of discrepancies between data and the theoretical predictions based on NN potentials only, revealed that 3NFs are unavoidable to explain the cross-sections and polarization data, especially the analyzing powers and spin transfer coefficients [11–24]. Generally, the discrepancies between data and pure NN potential predictions increase with the energy of the 3N system. However, also here the discrepancies which remain after inclusion of the TM or Urbana IX 3NFs indicate the incompleteness of these models.

With increasing energy it is possible that also effects due to relativity become more and more important. Up

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to very recently they have always been neglected in the analysis of 3N scattering processes. This has changed now. A relativistic framework in the instant form of dynamics has been applied in [25]. It turned out that the angular distribution in elastic nucleon-deuteron (Nd) scattering is sensitive to relativistic effects only at backward angles and relativity moves theory closer to the data. The above-mentioned spin observables were less sensitive. A greater sensitivity to relativity, however, appears in certain Nd breakup configurations [26], which deserve to be tested experimentally. Thus effects due to relativity should not be misinterpreted as 3NF effects. Therefore relativity should also be included when analyzing higher-energy elastic scattering and breakup data.

In view of this unsettled situation it appears very helpful to further test the spin dependence of the available 3N forces and the possible occurrence of relativistic effects. We, therefore, examined a group of analyzing powers and spin-correlation coefficients in elastic pd scattering at the proton laboratory energies $E_p^{lab} = 135$ and 200 MeV. These data became available with the advent of storage rings with internal targets, which opened new possibilties for experiments with polarized targets and beams [27]. Using a vector and tensor polarized deuterium target at the IUCF Cooler and a polarized proton beam the following analyzing powers and spin correlations were measured: $A_{xx} - A_{yy}, A_y(d), A_y(p), A_{zz}, C_{xx,y} - C_{yy,y}, C_{x,x}, C_{xy,x},$ $C_{x,z}, C_{xz,y}, C_{y,y}, C_{yz,x}, C_{z,z}, and C_{zz,y}$ [28]. In this paper we would like to analyze this rich set of data.

In sect. 2 we briefly remind the reader of the dynamical input which enters into our Faddeev calculational scheme. Then in sect. 3 we confront the experimental data with theoretical calculations based on different dynamical ingredients: two-nucleon forces only, two- and three-nucleon forces, and adding relativity. We conclude in sect. 4.

2 Theory

Nucleon-deuteron elastic scattering with neutron and protons interacting through a NN potential V and through a 3NF V_4 is described in terms of a breakup operator T satisfying the Faddeev-type integral equation [29,12,30]

$$T = tP + (1 + tG_0)V_4^{(1)}(1 + P) + tPG_0T + (1 + tG_0)V_4^{(1)}(1 + P)G_0T.$$
 (1)

The two-nucleon (2N) *t*-matrix *t* results from the interaction *V* through the Lippmann-Schwinger equation. The permutation operator $P = P_{12}P_{23} + P_{13}P_{23}$ is given in terms of the transposition P_{ij} which interchanges nucleons *i* and *j*, and G_0 is the free 3N propagator. Finally, the operator $V_4^{(1)}$ appearing in eq. (1) is a part of the full 3NF $V_4 = V_4^{(1)} + V_4^{(2)} + V_4^{(3)}$ and is symmetric under the exchange of nucleons 2 and 3. For instance, in the case of the π - π exchange 3NF such a decomposition corresponds to the three possible choices of the nucleon which undergoes off-shell π -N scattering. It is understood that the operator T acts on the incoming state $|\phi\rangle = |\mathbf{q}_0\rangle |\phi_d\rangle$ which describes the free nucleon-deuteron motion with relative momentum \mathbf{q}_0 and the deuteron wave function $|\phi_d\rangle$. The physical picture underlying eq. (1) is revealed after iterations which lead to a multiple scattering series for T.

The transition operator U for elastic Nd scattering is given in terms of T by [29, 12, 30]

$$U = PG_0^{-1} + PT + V_4^{(1)}(1+P) + V_4^{(1)}(1+P)G_0T.$$
 (2)

We solve eq. (1) in momentum space using a partialwave decomposition for each total angular momentum Jand parity of the 3N system. To achieve converged results a sufficiently high number of partial waves is used. Calculations with and without 3NF are performed including all 3N partial-wave states with the total two-body angular momenta $j \leq 5$. Equation (1) is usually solved for J up to 25/2. The 3NF is taken into account for J up to 13/2 which is sufficient because of the shorter range of the 3NF compared to the NN interaction. In all calculations we neglect the total isospin T = 3/2 contribution in the state ${}^{1}S_{0}$ and use in this state the np form of the NN interaction. Such a restriction to the np force for the ${}^{1}S_{0}$ state does not have a significant effect on the analyzing powers and spin correlation coefficients at our energies.

As the NN interactions we use the modern NN potentials AV18, CD Bonn, NijmI and NijmII and combine each of them with the 2π -exchange TM99 3NF [31], adjusting the cut-off parameter of strong form factors in the TM99 force individually to get the experimental triton binding energy. The resulting cut-offs for these potentials are, respectively, 5.215, 4.856, 5.120, and 5.072 (in units of the pion mass m_{π}). In addition, the AV18 potential is also combined with the Urbana IX 3NF. In all calculations we neglect the long-ranged Coulomb force acting between two protons when proton-deuteron scattering is considered. At the energies of the present paper effects of this force should be small.

In order to estimate relativistic effects on the studied spin observables at our energies we solve eq. (1) including relativistic features. This encompasses relativistic kinematics and boost effects. We follow the approach presented in [25]. For the convenience of the reader the main dynamical ingredients are shortly described in the following.

The formal structure of the nonrelativistic and relativistic Faddeev equations is the same, only the ingredients change. In this study of relativistic effects we drop the 3NF. The relativistic kinetic energy H_0 of three equalmass (m) nucleons in their 3N c.m. system can be written as [25]

$$H_0 = \sqrt{(2\omega(\mathbf{k}))^2 + \mathbf{q}^2} + \sqrt{m^2 + \mathbf{q}^2},$$
 (3)

where $2\omega(\mathbf{k}) \equiv 2\sqrt{m^2 + \mathbf{k}^2}$, and \mathbf{q} is the momentum of the third nucleon whereas $-\mathbf{q}$ is the total momentum of the chosen two-body subsystem. In that two-body subsystem the two nucleons have momenta \mathbf{k} and $-\mathbf{k}$, respectively. They are connected to the individual nucleon momenta in an arbitrary frame by a free Lorentz transformation.

The full 3N Hamiltonian contains besides H_0 the sum of pair interactions $V(\mathbf{q})$. They have the form

$$V(\mathbf{q}) \equiv \sqrt{(2\omega(\mathbf{k}) + v)^2 + \mathbf{q}^2} - \sqrt{(2\omega(\mathbf{k}))^2 + \mathbf{q}^2}, \qquad (4)$$

where v is the potential defined in the 2N c.m. system.

Equations (3) and (4) define new ingredients, which enter eq. (1): the boosted t-operator which satisfies the relativistic 2N Lippmann-Schwinger equation

$$t(\mathbf{k}, \mathbf{k}'; \mathbf{q}) = V(\mathbf{k}, \mathbf{k}'; \mathbf{q}) + \int d^3 k'' \frac{V(\mathbf{k}, \mathbf{k}''; \mathbf{q}) t(\mathbf{k}'', \mathbf{k}'; \mathbf{q})}{\sqrt{(2\omega(\mathbf{k}')^2 + \mathbf{q}^2 - \sqrt{(2\omega(\mathbf{k}'')^2 + \mathbf{q}^2 + i\epsilon)}}$$
(5)

and the relativistic 3N propagator

$$G_0 = \frac{1}{E + i\epsilon - H_0}.$$
(6)

For the technical performance, the momentum space partial-wave decomposition and the corresponding expression of the matrix elements of the permutation operator P we refer to [25]. There also details are given on how to solve the relativistic Faddeev equations.

As dynamical input for our relativistic calculations we use relativistic interactions v generated from the nonrelativistic NN potentials AV18 and CD Bonn according to the analytical prescription of ref. [32]. Such relativistic NN interactions are exactly on-shell equivalent to the underlying nonrelativistic potentials. To avoid a rather complicated calculation of the matrix element $V(\mathbf{k}, \mathbf{k}'; \mathbf{q})$ for the boosted potential (see eq. (4) of ref. [33]) we restrict ourselves to the leading-order term in a q/ω and v/ω expansion

$$V(\mathbf{k}, \mathbf{k}'; \mathbf{q}) = v(\mathbf{k}, \mathbf{k}') \left[1 - \frac{\mathbf{q}^2}{8\sqrt{m^2 + \mathbf{k}^2}\sqrt{m^2 + (\mathbf{k}')^2}} \right].$$
(7)

As was checked in [25], such an approximation is acceptable even for the strongest boosts appearing in the present paper.

In our relativistic calculations we also neglect the Wigner spin rotations. In [25] we performed a restricted study allowing only j < 2 angular momenta and found that for elastic spin observables the Wigner spin rotation effects are rather small. Presently, their full inclusion surpasses our available computer resources.

3 Comparison of theory and data

We use four different NN forces, which describe the NN data set equally well. The Nijmegen and CD Bonn potentials are of one-boson exchange type, whereas AV18 is more phenomenological. All potentials require for the fine tuning to the NN data set about 40–45 parameters. As we shall see, their predictions for 3N observables do not agree but span bands for the different observables. Now adding our two 3NF models will shift the bands. One can speak



Fig. 1. (Color online) The deuteron $A_y(d)$ and neutron $A_y(n)$ vector analyzing powers in elastic neutron-deuteron (nd) scattering at incident neutron energy $E_n^{lab} = 135$ MeV (first row) and 200 MeV (second row). Open circles are the pd data of ref. [28]. The light-shaded (cyan) bands contain the NN force predictions (AV18, CD Bonn, NijmI and II), and the dark-shaded (magenta) bands contain the combinations of the NN+TM99 3NF predictions as described in the text. The solid curve is the AV18+Urbana IX 3NF prediction.



Fig. 2. (Color online) The same as in fig. 1 but for the tensor analyzing powers $A_{xx} - A_{yy}$ and A_{zz} .

of significant 3NF model effects only if the two types of bands clearly separate. The aim of our analysis is to study whether such significant 3NF effects are present and, if yes, in which observables. Then, we are also interested to know whether the two 3NF models investigated improve the description of the data.

We show in figs. 1-3 the experimental data for the analyzing powers $A_y(d)$, $A_y(n)$, $A_{xx} - A_{yy}$, A_{zz} , and A_{xz} at 135 and 200 MeV. They are compared to the nonrelativistic NN force predictions only, which span the lightshaded bands. Qualitatively the bands follow the trend of the data, but serious discrepancies are present. They



Fig. 3. (Color online) The same as in fig. 1 but for the tensor analyzing power A_{xz} and the spin correlation coefficients $C_{x,z}$ and $C_{z,z}$. For the spin correlation coefficients $C_{x,z}$ and $C_{z,z}$ only the case of the incident neutron energy $E_n^{lab} = 135$ MeV is displayed.



Fig. 4. (Color online) The same as in fig. 1 but for the spin correlation coefficients $C_{zz,y}$ and $C_{y,y}$.

stick out for $A_{xx} - A_{yy}$ at 200 MeV and for A_{zz} at both energies.

Now adding the TM99 3NF to the four NN forces produces the darker-shaded bands. Nearly always the two types of bands are clearly separated and therefore 3NF model effects are present. But it is only for certain angular regions and for one or the other energy that one can see an improvement. Also the opposite effect occurs, namely that the agreement with the data is worsened.

We also added the special combination AV18 + Urbana IX prediction as a single solid line. Except for A_{xz} , where that prediction is close to the data, it always lies in the neighborhood of the TM 3NF model results.

We have to conclude that the serious discrepancies between the theoretical results and the data using the



Fig. 5. (Color online) The same as in fig. 1 but for the spin correlation coefficients $C_{xz,y}$ and $C_{x,x}$.



Fig. 6. (Color online) The same as in fig. 1 but for the spin correlation coefficients $C_{xy,x}$ and $C_{z,x}$.

NN force models only are not systematically improved by adding the two 3NF models TM99 and Urbana IX.

The spin correlation coefficients $C_{xx,y} - C_{yy,y}$, $C_{x,x}$, $C_{xy,x}$, $C_{x,z}$, $C_{xz,y}$, $C_{y,y}$, $C_{yz,x}$, $C_{z,x}$, $C_{z,z}$, and $C_{zz,y}$ are displayed in figs. 3-7. The presentation is the same as before. Here for $C_{zz,y}$, $C_{y,y}$, and $C_{xz,y}$ the 3NF model effects shift theory closer to the data. But for $C_{x,x}$, $C_{xy,x}$, $C_{z,z}$, and $C_{x,z}$ this is just opposite: the shifts by the 3NF models move theory away from the data. For the spin-observable $C_{z,x}$ adding the 3NF models leads to an improvement or disimprovement depending on the angular region. Finally, for $C_{xx,y} - C_{yy,y}$ and $C_{yz,x}$ the 3NF model effects are small and there is a fair agreement with the data. Again, in most cases the combination AV18 + Urbana IX does not deviate significantly from the TM99 predictions with the case of $C_{x,z}$ the AV18 + Urbana IX combination is only weakly affected by the addition of the 3NF and thus close to the data.



Fig. 7. (Color online) The same as in fig. 1 but for the spin correlation coefficients $C_{yz,x}$ and $C_{xx,y} - C_{yy,y}$.



Fig. 8. The deuteron vector $A_y(d)$ and tensor $A_{xx} - A_{yy}$ analyzing powers in elastic nd scattering at the incident neutron energy $E_n^{lab} = 135$ MeV (first row) and 200 MeV (second row). Open circles are the pd data of ref. [28]. The solid curve is the result of the nonrelativistic Faddeev calculation with the AV18 potential. The relativistic prediction (see text) with the AV18 interaction is shown by the dash-dotted curve. The nonrelativistic and relativistic predictions based on the CD Bonn potential are given by the dashed and dotted curves, respectively.

Again we have to conclude that significant 3NF model effects are present, but no general improvement results. On the contrary, significant deterioration is also present. In view of the promising results for the spectra of light nuclei [9,10], where only the addition of the IL2 3N force on top of Urbana IX lead to a significantly improved agreement with the data, our findings are presumably not too surprising. Besides $\pi N s$ -wave scattering for the 2π -exchange 3NF, which is included in the TM 3NF, the new structures in IL2 are three-pion exchange ring diagrams with only one Δ at a time in the intermediate states. It would be very interesting to probe that new spin-isospin



Fig. 9. The same as in fig. 8 but for the spin correlation coefficients $C_{y,y}$ and $C_{xy,x}$.

structure also in Nd scattering. Also the effective field theory approach based on chiral symmetry points to further 3N force structures [34]. Thus already at next-to-next-toleading order of chiral expansion that approach predicts besides the 2π -exchange 3N force two more structures: a one-pion exchange combined with a NN contact force and a pure contact 3NF. Thus in a meson exchange picture additional 3N force models like $\pi - \rho$, $\rho - \rho$, ... exchanges [35] might be envisaged.

It remains to investigate relativistic effects. We performed that study for NN forces only and this for two different cases, CD Bonn and AV18. The effects are always very small and we display only a few of the "strongest" examples in figs. 8 and 9. This is similar to results shown in [25] where for spin observables no drastic changes due to relativity have been found. We have to conclude that for the relativistic scenario used in our study the relativistic effects are quite insignificant and cannot remedy the serious defects produced by the 3NF models under discussion.

4 Conclusion

Our Faddeev calculational analysis of a set of analyzing powers and spin correlation coefficients in elastic pd scattering at 135 and 200 MeV nucleon laboratory energy revealed:

NN force predictions alone using the standard highprecision potentials are insufficient to describe the data quantitatively, though qualitatively most often the angular trends are respected.

When the TM99 3NF model, individually adjusted for the different NN forces to the ³H binding energy, is added no systematic improvement results. On the contrary, for some observables, the description is even deteriorated. One has to conclude that significant parts of the 3NF are missing. The situation is not better when using the other well-known combination of the AV18 NN and the Urbana IX 3NF forces.

These discrepancies cannot be removed by adding relativistic effects of a specific choice. We used an instant form relativistic approach which encompasses relativistic kinematics and boost corrections to the NN forces. The relativistic effects turned out to be quite insignificant similarly as found in [25].

In view of the drastic discrepancies it is mandatory to add further 3N force structures on top of TM99 or Urbana IX. The successes found in [9,10] for the spectra of light nuclei using the IL2 3N force in addition suggests that also in Nd scattering this might improve the description of the data and would possibly help to strengthen the role played by such a 3N force model.

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